

Postulates, Theorems, and Corollaries Chapter 8

Thm. 8-1-1 The altitude to the hypotenuse of a right triangle forms two triangles that are similar to each other and to the original triangle.

Cor. 8-1-2 Geometric Means Corollary The length of the altitude to the hypotenuse of a right triangle is the geometric mean of the lengths of the two segments of the hypotenuse.

Cor. 8-1-3 Geometric Means Corollary The length of a leg of a right triangle is the geometric mean of the lengths of the hypotenuse and the segment of the hypotenuse adjacent to that leg.

Thm. 8-5-1 The Law of Sines For any $\triangle ABC$ with side lengths a , b , and c , $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.

Thm. 8-5-2 The Law of Cosines For any $\triangle ABC$ with sides a , b , and c , $a^2 = b^2 + c^2 - 2bccos A$, $b^2 = a^2 + c^2 - 2accos B$, and $c^2 = a^2 + b^2 - 2abcos C$.

Postulates, Theorems, and Corollaries Chapter 7

Post. 7-3-1 Angle-Angle (AA) Similarity Postulate If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.

Thm. 7-3-2 Side-Side-Side (SSS) Similarity Theorem If the three sides of one triangle are proportional to the three corresponding sides of another triangle, then the triangles are similar.

Thm. 7-3-3 Side-Angle-Side (SAS) Similarity Theorem If two sides of one triangle are proportional to two sides of another triangle and their included angles are congruent, then the triangles are similar.

Thm. 7-4-1 Triangle Proportionality Theorem If a line parallel to a side of a triangle intersects the other two sides, then it divides those sides proportionally.

Thm. 7-4-2 Converse of the Triangle Proportionality Theorem If a line divides two sides of a triangle proportionally, then it is parallel to the third side.

Cor. 7-4-3 Two-Transversal Proportionality Corollary If three or more parallel lines intersect two transversals, then they divide the transversals proportionally.

Thm. 7-4-4 Triangle Angle Bisector Theorem An angle bisector of a triangle divides the opposite side into two segments whose lengths are proportional to the lengths of the other two sides.

Thm. 7-5-1 Proportional Perimeters and Areas Theorem If the similarity ratio of two similar figures is $\frac{a}{b}$, then the ratio of their perimeters is $\frac{a}{b}$, and the ratio of their areas is $\frac{a^2}{b^2}$ or $\left(\frac{a}{b}\right)^2$.

Properties of Similarity

Reflexive Property of Similarity

$\triangle ABC \sim \triangle ABC$ (Reflex. Prop. of \sim)

Symmetric Property of Similarity

If $\triangle ABC \sim \triangle DEF$, then $\triangle DEF \sim \triangle ABC$. (Sym. Prop. of \sim)

Transitive Property of Similarity

If $\triangle ABC \sim \triangle DEF$ and $\triangle DEF \sim \triangle XYZ$, then $\triangle ABC \sim \triangle XYZ$.
(Trans. Prop. of \sim)

Postulates, Theorems, and Corollaries Chapter 5



- Thm. 5-1-1 Perpendicular Bisector Theorem** If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.
- Thm. 5-1-2 Converse of the Perpendicular Bisector Theorem** If a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.
- Thm. 5-1-3 Angle Bisector Theorem** If a point is on the bisector of an angle, then it is equidistant from the sides of the angle.
- Thm. 5-1-4 Converse of the Angle Bisector Theorem** If a point in the interior of an angle is equidistant from the sides of the angle, then it is on the bisector of the angle.
- Thm. 5-2-1 Circumcenter Theorem** The circumcenter of a triangle is equidistant from the vertices of the triangle.
- Thm. 5-2-2 Incenter Theorem** The incenter of a triangle is equidistant from the sides of the triangle.
- Thm. 5-3-1 Centroid Theorem** The centroid of a triangle is located $\frac{2}{3}$ of the distance from each vertex to the midpoint of the opposite side.
- Thm. 5-4-1 Triangle Midsegment Theorem** A midsegment of a triangle is parallel to a side of a triangle, and its length is half the length of that side.
- Thm. 5-5-1** If two sides of a triangle are not congruent, then the larger angle is opposite the longer side.
- Thm. 5-5-2** If two angles of a triangle are not congruent, then the longer side is opposite the larger angle.
- Thm. 5-5-3 Triangle Inequality Theorem** The sum of any two side lengths of a triangle is greater than the third side length.
- Thm. 5-6-1 Hinge Theorem** If two sides of one triangle are congruent to two sides of another triangle and the included angles are not congruent, then the longer third side is across from the larger included angle.
- Thm. 5-6-2 Converse of the Hinge Theorem** If two sides of one triangle are congruent to two sides of another triangle and the third sides are not congruent, then the larger included angle is across from the longer third side.
- Thm. 5-7-1 Converse of the Pythagorean Theorem** If the sum of the squares of the lengths of two sides of a triangle is equal to the square of the length of the third side, then the triangle is a right triangle.
- Thm. 5-7-2 Pythagorean Inequalities Theorem** In $\triangle ABC$, c is the length of the longest side. If $c^2 > a^2 + b^2$, then $\triangle ABC$ is an obtuse triangle. If $c^2 < a^2 + b^2$, then $\triangle ABC$ is an acute triangle.
- Thm. 5-8-1 45°-45°-90° Triangle Theorem** In a 45°-45°-90° triangle, both legs are congruent, and the length of the hypotenuse is the length of a leg times $\sqrt{2}$.
- Thm. 5-8-2 30°-60°-90° Triangle Theorem** In a 30°-60°-90° triangle, the length of the hypotenuse is 2 times the length of the shorter leg, and the length of the longer leg is the length of the shorter leg times $\sqrt{3}$.

Postulates, Theorems, and Corollaries Chapter 4



Thm. 4-3-1 Triangle Sum Theorem The sum of the angle measures of a triangle is 180° .

Cor. 4-3-2 The acute angles of a right triangle are complementary.

Cor. 4-3-3 The measure of each angle of an equiangular triangle is 60° .

Thm. 4-3-4 Exterior Angle Theorem The measure of an exterior angle of a triangle is equal to the sum of the measures of its remote interior angles.

Thm. 4-3-5 Third Angles Theorem If two angles of one triangle are congruent to two angles of another triangle, then the third pair of angles are congruent.

Post. 4-5-1 Side-Side-Side (SSS) Congruence Postulate If three sides of one triangle are congruent to three sides of another triangle, then the triangles are congruent.

Post. 4-5-2 Side-Angle-Side (SAS) Congruence Postulate If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent.

Post. 4-6-1 Angle-Side-Angle (ASA) Congruence Postulate If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.

Thm. 4-6-2 Angle-Angle-Side (AAS) Congruence Theorem If two angles and a nonincluded side of one triangle are congruent to the corresponding angles and nonincluded side of another triangle, then the triangles are congruent.

Thm. 4-6-3 Hypotenuse-Leg (HL) Congruence Theorem If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of another right triangle, then the triangles are congruent.

Thm. 4-9-1 Isosceles Triangle Theorem If two sides of a triangle are congruent, then the angles opposite the sides are congruent.

Thm. 4-9-2 Converse of the Isosceles Triangle Theorem If two angles of a triangle are congruent, then the sides opposite those angles are congruent.

Cor. 4-9-3 If a triangle is equilateral, then it is equiangular.

Cor. 4-9-4 If a triangle is equiangular, then it is equilateral.

Postulates, Theorems, and Corollaries Chapter 3



Post. 3-2-1 Corresponding Angles Postulate If two parallel lines are cut by a transversal, then the pairs of corresponding angles are congruent.

Thm. 3-2-2 Alternate Interior Angles Theorem If two parallel lines are cut by a transversal, then the pairs of alternate interior angles are congruent.

Thm. 3-2-3 Alternate Exterior Angles Theorem If two parallel lines are cut by a transversal, then the two pairs of alternate exterior angles are congruent.

Thm. 3-2-4 Same-Side Interior Angles Theorem If two parallel lines are cut by a transversal, then the two pairs of same-side interior angles are supplementary.

Post. 3-3-1 Converse of the Corresponding Angles Postulate If two coplanar lines are cut by a transversal so that a pair of corresponding angles are congruent, then the two lines are parallel.

Post. 3-3-2 Parallel Postulate Through a point P not on line ℓ , there is exactly one line parallel to ℓ .

Thm. 3-3-3 Converse of the Alternate Interior Angles Theorem If two coplanar lines are cut by a transversal so that a pair of alternate interior angles are congruent, then the two lines are parallel.

Thm. 3-3-4 Converse of the Alternate Exterior Angles Theorem If two coplanar lines are cut by a transversal so that a pair of alternate exterior angles are congruent, then the two lines are parallel.

Thm. 3-3-5 Converse of the Same-Side Interior Angles Theorem If two coplanar lines are cut by a transversal so that a pair of same-side interior angles are supplementary, then the two lines are parallel.

Thm. 3-4-1 If two intersecting lines form a linear pair of congruent angles, then the lines are perpendicular.

Thm. 3-4-2 Perpendicular Transversal Theorem In a plane, if a transversal is perpendicular to one of two parallel lines, then it is perpendicular to the other line.

Thm. 3-4-3 If two coplanar lines are perpendicular to the same line, then the two lines are parallel to each other.

Thm. 3-5-1 Parallel Lines Theorem In a coordinate plane, two nonvertical lines are parallel if and only if they have the same slope. Any two vertical lines are parallel.

Thm. 3-5-2 Perpendicular Lines Theorem In a coordinate plane, two nonvertical lines are perpendicular if and only if the product of their slopes is -1 . Vertical and horizontal lines are perpendicular.

Postulates, Theorems, and Corollaries Chapter 2



Thm. 2-6-1 Linear Pair Theorem If two angles form a linear pair, then they are supplementary.

Thm. 2-6-2 Congruent Supplements Theorem If two angles are supplementary to the same angle (or to two congruent angles), then the two angles are congruent.

Thm. 2-6-3 Right Angle Congruence Theorem All right angles are congruent.

Thm. 2-6-4 Congruent Complements Theorem If two angles are complementary to the same angle (or to two congruent angles), then the two angles are congruent.

Thm. 2-7-1 Common Segments Theorem Given collinear points A , B , C , and D arranged as shown, if $\overline{AB} \cong \overline{CD}$, then $\overline{AC} \cong \overline{BD}$.



Thm. 2-7-2 Vertical Angles Theorem Vertical angles are congruent.

Thm. 2-7-3 If two congruent angles are supplementary, then each angle is a right angle.

Postulates, Theorems, and Corollaries Chapter 1



Post. 1-1-1 Through any two points there is exactly one line.

Post. 1-1-2 Through any three noncollinear points there is exactly one plane containing them.

Post. 1-1-3 If two points lie in a plane, then the line containing those points lies in the plane.

Post. 1-1-4 If two lines intersect, then they intersect in exactly one point.

Post. 1-1-5 If two planes intersect, then they intersect in exactly one line.

Post. 1-2-1 Ruler Postulate The points on a line can be put into a one-to-one correspondence with the real numbers.

Post. 1-2-2 Segment Addition Postulate If B is between A and C , then $AB + BC = AC$.

Post. 1-3-1 Protractor Postulate Given \overleftrightarrow{AB} and a point O on \overleftrightarrow{AB} , all rays that can be drawn from O can be put into a one-to-one correspondence with the real numbers from 0 to 180.

Post. 1-3-2 Angle Addition Postulate If S is in the interior of $\angle PQR$, then $m\angle PQS + m\angle SQR = m\angle PQR$.

Thm. 1-6-1 Pythagorean Theorem In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.